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Semantic Analysis of the Concrete Pictorial Abstract Approach in Understanding Mathematical Concepts

Yandika Nugraha[1], Mulhamah[2*]
[1]Mathematics Education, Universitas Islam Negeri Mataram, Indonesia.
Author Corresponding: mulhamah@uinmataram.ac.id*

ABSTRACT

Mathematics is not only procedural, but also conceptual. When students are able to interpret mathematical concepts, then students have freedom and are not just stuck to standard procedures from rote memorization. Therefore, a concrete pictorial abstract approach is needed as a bridge to understanding mathematical concepts. In this study, we investigated prospective teachers in making sense of concrete, pictorial, and abstract processes in understanding mathematical concepts. The subjects of this research were 4th semester students taking school mathematics courses I. Data collection was carried out by means of task-based interviews which were carried out after the learning process ended. The data analysis and interpretation technique uses a semantic framework. The results of the data analysis found two categories in the semantic CPA process, namely full and partial use of CPA. In the full semantic CPA process, students are able to use concrete, pictorial and abstract approaches in solving problems and interpreting mathematical concepts. In the partial semantic CPA process, students use concrete and pictorial stages to help illustrate the problem. Apart from that, students only use an abstract approach in solving problems. When students are able to fully understand the use of CPA, it can be concluded that the concept of mathematics has been internalized.

Keywords: Semantic; concrete pictorial abstract; understanding of mathematical concepts

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1. INTRODUCTION

Mathematics is widely perceived as a difficult subject by students across primary, secondary, and higher education levels. This perception stems from several factors, most notably a lack of conceptual grasp and a reliance on purely procedural understanding. Consequently, students often experience confusion when problem contexts vary. If this issue persists, it solidifies the stigma that mathematics is inherently difficult. To break this cycle, a fundamental requirement is for students to achieve conceptual rather than merely procedural understanding during the problem-solving process.

Conceptual understanding has been internationally emphasized as a cornerstone of mathematics education (Kim, 2020). This level of understanding extends beyond rote memorization; it must be deep enough to ensure that mathematical concepts become meaningful (Fernández et al., 2019). This focus on meaningfulness has given rise to the study of semantics in the interpretation of mathematical concepts.

Meaningfulness is intrinsically linked to the term semantics (Birdsong et al., 1995; Easdown, 2009). In this context, it is understood as the relationship between mathematical concepts and their underlying symbols (Fonseca, 2020; Greiner-Petter et al., 2020). While articulating the meaning of a mathematical concept is often challenging (Fernández et al., 2019), the ability to interpret and apply information simultaneously can significantly facilitate problem-solving (Zhong, 2017). Assigning meaning to mathematical material must involve both procedural and conceptual dimensions. When students can internalize the semantics of a concept, they gain cognitive flexibility and are no longer restricted to rigid, memorized procedures.

Instructional practices that prioritize meaningful conceptual understanding are essential. Consequently, there is a demand for teachers who can master pedagogy centered on meaningful conceptualization rather than rote procedures. As the vanguard in producing qualified educators, higher education institutions are expected to support this transition.

Understanding the semantic meaning of basic mathematical concepts is vital (Walle et al., 2015). To make mathematics meaningful, an appropriate pedagogical approach is required, particularly for foundational topics. One such method is the concrete-to-abstract approach. Tall (2008) explains that conceptual understanding begins with concrete objects, involving perception and reflection. These tangible objects are initially observed and felt in the physical world before being abstracted. Furthermore, Roth and Hwang (2006) state that the transition from concrete to abstract represents a practical understanding of a concept during the initial stages of learning.

Yoong (2015) and Kim (2020) utilize the term Concrete-Pictorial-Abstract (CPA), where the CPA sequence is grounded in Bruner's three modes of representation: enactive, iconic, and symbolic. This approach aligns with the cognitive development stages identified in various psychological theories and serves as a refinement of the earlier Concrete-Representational-Abstract (CRA) model. Another term used by Walle et al. (2015) is Concrete-Semiconcrete-Abstract (CSA). These approaches emphasize mastering mathematical concepts through three stages: concrete, pictorial (representational or semiconcrete), and abstract. Utilizing this progression can enhance students' ability to grasp mathematical concepts effectively (Roth & Hwang, 2006). This approach emphasizes active, hands-on learning, allowing students to deeply internalize mathematical concepts from the earliest stages through the development of abstract thinking.

Extant literature has explored semantics and the CPA approach independently. However, the researcher has yet to find a study that specifically integrates these two domains. Pape (2004) and Prayitno et al. (2020) elucidate the role of semantic processing in understanding mathematical concepts and problem-solving. The progression of understanding from concrete to abstract is supported by the theories of Roth & Hwang (2006), Kamina & Iyer (2009), Walle et al. (2015), and Kim (2020). Therefore, the researcher intends to conduct a study analyzing the semantics of the Concrete-Pictorial-Abstract approach in conceptual understanding. The researcher hypothesizes the emergence of a new theoretical phase, termed internalization, which occurs after the meaningful interpretation of a mathematical concept.

Based on preliminary observations within the Mathematics Education Department, the researcher identified that many students possess only procedural knowledge without a meaningful grasp of concepts. This was evident in an assignment to develop instructional aids for a Learning Media course; students focused on the procedural construction of the media without understanding the underlying mathematical principles. Consequently, they were unable to explain the concepts behind the tools they created, despite the material being limited to primary and secondary school mathematics. This deficiency may lead to a lack of comprehension of more complex topics in subsequent semesters. Furthermore, it could negatively impact their Teaching Practicum and their future performance as professional educators. Given that these students are the next generation of teachers, this issue requires immediate attention. To address this, the researcher will conduct a study involving fourth-semester students enrolled in the "School Mathematics 1" class, which focuses on fundamental mathematical content.

2. METHODS

This study adopts a qualitative research paradigm. According to Creswell (2012), a qualitative approach is essential for exploring a problem and investigating it in depth. This research was

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conducted to understand conceptual grasp by referring to the semantic process of the Concrete-Pictorial-Abstract (CPA) sequence.

This study is classified as exploratory research, in which the researcher seeks to investigate a novel and compelling phenomenon (Elman et al., 2020) related to the semantic processes of CPA in understanding mathematical concepts. The researcher conducted an in-depth observation of the participants' thoughts, actions, writings, drawings, and verbal expressions. These data were subsequently presented based on actual conditions and analyzed to reveal the semantic processes of the CPA approach in mathematical conceptualization.

The research was conducted at the Mathematics Education Department. The participants consisted of all fourth-semester students enrolled in the "School Mathematics I" and "Mathematics Learning Media" courses.

The data in this study were categorized into two primary types: document data and interview data. Document data comprised the results of mathematical conceptual understanding tests. Interview data were obtained through interviews with the subjects, which were subsequently presented in the form of interview transcripts. The data sources were derived directly from the students to uncover the semantic processes of the CPA approach in understanding mathematical concepts.

The data analysis process was conducted concurrently with data collection. In addition to gathering data, the researcher performed continuous data analysis using a framework adapted from Pape (2004) and Prayitno et al. (2022). The framework used to analyze the CPA semantic process is presented in the following table:

Table 1. Framework for Analyzing the CPA Semantic Process

Stage	Indicators
Sorting	Sorting and expressing the problem situation.
Identification	Identifying the contextual structure of the problem.
Argument Formulation	Establishing relationships, constructing equations, and interpreting meanings.
Verification	Providing evidence and explanations for calculation processes and interpreting meanings.

The data analysis process is further elaborated as follows:

1. Preparation and Review: Examining all collected data, including test results, conceptual understanding assessments, interview transcripts, and field notes.
2. Data Reduction: Selecting, focusing, simplifying, and abstracting the data.
3. Data Unitizing: Organizing data into units to facilitate interpretation and analysis.
4. Conclusion Drawing: Formulating conclusions regarding the semantic processes of the CPA approach in understanding mathematical concepts.

During the analysis, the researcher categorized the findings into two groups: Full CPA and Partial CPA. The first category includes subjects capable of utilizing the CPA approach comprehensively to understand mathematical concepts. The second category includes subjects who do not fully utilize all CPA stages. This distinction is crucial for an in-depth analysis because even if the final result is correct, the underlying cognitive processes may differ.

To ensure that the qualitative data obtained were accurate and credible, the researcher performed data trustworthiness tests through triangulation. This study employed methodological triangulation, where the researcher conducted a deep review of both the test result sheets and interview transcripts. Detailed field notes were maintained to record significant events during the research process, which served to support and validate the findings. Furthermore, the researcher conducted peer debriefing by discussing findings and data interpretations with colleagues. These peers included not only members of the research team but also other researchers with relevant expertise to obtain critical feedback and suggestions, thereby enhancing the quality of the data.

3. RESULT AND DISCUSSION

Based on data collected from four classes consisting of 75 students from the 2022 cohort, the researcher identified two distinct categories. The first category comprises subjects who utilize the Concrete-Pictorial-Abstract (CPA) approach to understand mathematical concepts, hereafter referred to as Full CPA. The second category includes subjects who do not fully employ all stages of the CPA sequence, hereafter referred to as Partial CPA. The analysis revealed that out of 75 students, only 6 fell into the Full CPA category, while 68 belonged to the Partial CPA category.

3.1 Full CPA Semantic Category

To analyze this category, the researcher employed a semantic framework consisting of five stages: sorting, identification, argument formulation, verification, and conclusion drawing. The Full CPA category is characterized by the subject's ability to navigate all three CPA stages (Concrete, Pictorial, and Abstract) when solving problems. This category is represented by three subjects: Fb, Wr, and Rz. The following table details their semantic processes:

Table 2. Semantic Analysis of Full CPA Subjects

Stage	Subject Fb	Subject Wr	Subject Rz
Sorting	Successfully sorted information from the problem text to be represented through concrete, pictorial, and abstract media.	Demonstrated the ability to sort problem information for use across concrete, pictorial, and abstract representations.	Successfully identified and sorted information to be processed via concrete, pictorial, and abstract stages.
	Concrete: Planned a simulation involving students and real objects.	Concrete: Interpreted the problem by mapping real-world objects into easily understood items.	Concrete: Designed a problem-solving process involving direct classroom activities with provided objects.
	Pictorial: Planned the use of visual representations through drawings.	Pictorial: Planned visual representations in the form of sketches/images.	Pictorial: Planned the use of visual imagery.
	Abstract: Planned the use of symbols and numerical values.	Abstract: Planned the use of mathematical symbols and numbers.	Abstract: Planned the use of symbols and numbers.
Identification	Concrete: Defined key terms by transforming the problem into instructional steps for simulation.	Concrete: Defined key terms by breaking the problem into a three-step command sequence.	Concrete: Defined key terms by creating instructions to support learning simulations.
	Pictorial: Interpreted key terms by manipulating real objects into geometric shapes (circles and squares).	Pictorial: Interpreted key terms by creating direct pictorial representations of	Pictorial: Interpreted key terms by manipulating real forms into block representations.

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		trees (mango and orange).	
	Abstract: Assigned meaning to key terms by writing mathematical symbols.	Abstract: Identified key terms by utilizing formal symbols.	Abstract: Defined key terms through symbolic notation.
Argument Formulation	Concrete: Formulated relations between quantities (object vs. price).	Concrete: Established relations between contextual objects, direct objects, and fruit quantities.	Concrete: Formulated relations between objects and their respective prices.
	Pictorial: Established relations between objects and manipulated visual representations.	Pictorial: Formulated arguments by relating objects to the total quantity of fruit.	Pictorial: Related objects to visually manipulated images.
	Abstract: Created algebraic models/equations using symbols or variables to represent relations.	Abstract: Formulated arguments by relating objects to quantities via numerical symbols.	Abstract: Constructed equations by assigning variables to quantities.
Verification	Concrete/Pictorial/Abstract: Provided calculation evidence and explanations through verbal sentences, drawings, and symbols respectively.	Concrete/Pictorial/Abstract: Verified results by providing step-by-step evidence using descriptions, images, and symbols.	Concrete/Pictorial/Abstract: Confirmed the solution by providing systematic evidence through concrete, pictorial, and symbolic means.
Conclusion	Provided final answers and interpreted them within the problem context across all three CPA stages.	Drew conclusions by providing final answers validated through concrete, pictorial, and symbolic aids.	Provided final answers, interpreting the abstract concept based on its concrete and pictorial foundations.

Discussion of Full CPA

The Concrete-Pictorial-Abstract (CPA) approach is a instructional sequence where students are introduced to a concept through concrete objects, followed by pictorial representations, and finally abstracted into symbolic forms. This process aligns with children's cognitive development stages. This is consistent with Bruner (1966), who argued that understanding abstract concepts is optimized when students first grasp concrete forms and visual representations. While not all learning must strictly follow the CPA sequence especially when students can already mentally visualize objects educators often neglect the concrete and pictorial stages because mathematical problem-solving is predominantly symbolic.

Linking the CPA approach with semantic processes is vital. When educators understand CPA meaningfully, they can implement it across various mathematical concepts. This flexibility is essential in the context of the *Merdeka Belajar* Curriculum, which emphasizes differentiated instruction.

In this study, students in the Full CPA category demonstrated the ability to use all three stages either separately or simultaneously to find solutions. They recognized that abstract understanding is heavily dependent on concrete and pictorial foundations. Without these, mathematical concepts lack meaningfulness and fail to be properly internalized.

The researcher concludes that mathematical concepts are successfully internalized when the CPA approach is fully meaningfully interpreted. Internalization is the process by which an individual deeply masters mathematics rather than merely imitating procedures (Baker et al., 2020). This level of understanding allows students to discern when to use concrete, pictorial, or abstract methods or a combination of all three effectively.

3.2 Partial CPA Semantic Category

The majority of students (68 out of 75) fell into the Partial CPA category. This is primarily due to a cognitive bias toward abstract thinking. While the abstract stage in this category is similar to the Full CPA category, these students used concrete and pictorial stages merely as tools to illustrate or interpret the problem rather than as a foundational conceptual bridge.

In many cases, students bypassed concrete and pictorial representations entirely, relying solely on abstract procedures. This suggests that when these students become teachers, their instructional style may lean heavily toward abstraction. Relying exclusively on abstract methods risks a lack of deep conceptual understanding among their future pupils. As Handayani (2018) notes, a solid grasp of mathematical concepts is the foundation of teaching; without it, prospective teachers struggle to explain material effectively.

However, the Partial CPA process is not necessarily "incorrect." Cognitive development varies across age groups, and the complexity of the material plays a significant role. Students often struggle to apply concrete or pictorial approaches to high-level topics where such representations are less intuitive. A primary obstacle in mathematical problem-solving is the inability to link concepts to word-based problems, which rely heavily on deep conceptual interpretation (Putra et al., 2018).

4. CONCLUSION

Based on the data analysis, this study identifies two categories of semantic processes within the Concrete-Pictorial-Abstract (CPA) framework: Full CPA and Partial CPA. In the Full CPA semantic process, students demonstrate the ability to utilize concrete, pictorial, and abstract approaches comprehensively to solve problems and assign meaning to mathematical concepts. Conversely, in the Partial CPA process, students use concrete and pictorial stages merely as illustrative aids or rely exclusively on abstract approaches for problem-solving.

The semantic progression observed in students follows a five-stage sequence: sorting the problem, identifying its structure, formulating arguments, verifying results, and drawing conclusions. This study concludes that when students can fully interpret and navigate these CPA stages, mathematical concepts become deeply internalized.

The findings further reveal that the majority of students remain habituated to abstract thinking and encounter significant difficulties when required to engage with concrete objects or pictorial representations. Therefore, it is recommended that educators prioritize instructional materials that emphasize concrete and visual foundations, particularly in school mathematics courses. While this research focuses on uncovering the semantic analysis of CPA, further studies are needed to develop CPA-based instructional designs. Additionally, a comprehensive literature review regarding the integration of CPA-oriented content in school textbooks is necessary to support more effective mathematical learning.

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